EXERCISE – I | SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. If the vector $\vec{b}$ is collinear with the vector $\vec{a} = (2\sqrt{2}, -1, 4)$ and $|\vec{b}|$ = 10, then

(A) $\vec{a} \pm \vec{b} = 0$ (B) $\vec{a} \pm 2\vec{b} = 0$

(C) $2\vec{a} \pm \vec{b} = 0$ (D) none of these

2. The vertices of a triangle are A(1, 1, 2), B(4, 3, 1) and C(2, 3, 5). A vector representing the internal bisector of the angle A is

(A) $\hat{i} + \hat{j} + 2\hat{k}$ (B) $2\hat{i} - 2\hat{j} + \hat{k}$

(C) $2\hat{i} + 2\hat{j} - \hat{k}$ (D) $2\hat{i} + 2\hat{j} + \hat{k}$

3. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is

(A) $-\hat{i} + \hat{j} + 2\hat{k}$ (B) $3\hat{i} - \hat{j} + \hat{k}$

(C) $3\hat{i} + \hat{j} - \hat{k}$ (D) $\hat{i} - \hat{j} - \hat{k}$

4. If $|\vec{a}|$ = 5, $|\vec{a} - \vec{b}|$ = 8 and $|\vec{a} + \vec{b}|$ = 10, then $|\vec{b}|$ is equal to

(A) 1 (B) $\sqrt{57}$ (C) 3 (D) none of these

5. Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

(A) $\cos^{-1}\left(\frac{1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{2}\right)$

(C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{5}{9}\right)$

6. Vector $\vec{a}$ and $\vec{b}$ make an angle $\theta = \frac{2\pi}{3}$. if $|\vec{a}|$ = 1, $|\vec{b}|$ = 2, then $\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2$ is equal to

(A) 225 (B) 250 (C) 275 (D) 300

7. Unit vector perpendicular to the plane of the triangle ABC with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ of the vertices A, B, C is

(A) $\frac{(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})}{\Delta}$ (B) $\frac{(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})}{2\Delta}$

(C) $\frac{(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})}{4\Delta}$ (D) none of these

8. The value of $[(\vec{a} + 2\vec{b} - \vec{c}), (\vec{a} - \vec{b}), (\vec{a} - \vec{b} - \vec{c})]$ is equal to the box product

(A) $[\vec{a}\,\vec{b}\,\vec{c}]$ (B) $2[\vec{a}\,\vec{b}\,\vec{c}]$ (C) $3[\vec{a}\,\vec{b}\,\vec{c}]$ (D) $4[\vec{a}\,\vec{b}\,\vec{c}]$

9. If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a} \parallel (\vec{b} \times \vec{c})$, then $(\vec{a} \times \vec{b}).(\vec{a} \times \vec{c})$ is equal to

(A) $\vec{a}^2(\vec{b}.\vec{c})$ (B) $\vec{b}^2(\vec{a}.\vec{c})$ (C) $\vec{c}^2(\vec{a}.\vec{b})$ (D) none of these

10. Vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j}$

(A) $\frac{3}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ (B) $\frac{3}{\sqrt{6}}(2\hat{i} - \hat{j} - \hat{k})$

(C) $\frac{3}{\sqrt{114}}(8\hat{i} - 7\hat{j} - \hat{k})$ (D) $\frac{3}{\sqrt{114}}(-7\hat{i} + 8\hat{j} - \hat{k})$

11. Vector $\vec{x}$ satisfying the relation $\vec{A}.\vec{x} = c$ and $\vec{A} \times \vec{x} = \vec{B}$ is

(A) $\frac{c\vec{A} - (\vec{A}\times\vec{B})}{|\vec{A}|}$ (B) $\frac{c\vec{A} - (\vec{A}\times\vec{B})}{|\vec{A}|^2}$

(C) $\frac{c\vec{A} + (\vec{A}\times\vec{B})}{|\vec{A}|^2}$ (D) $\frac{c\vec{A} - 2(\vec{A}\times\vec{B})}{|\vec{A}|^2}$

12. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are linearly independent vectors, then which one of the following set of vectors is linearly dependent ?

(A) $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ (B) $\vec{a} - \vec{b}$, $\vec{b} - \vec{c}$, $\vec{c} - \vec{a}$

(C) $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ (D) none of these

13. If line $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\vec{r}.(3\hat{i} - 2\hat{j} - m\hat{k}) = 14$, then the value of m is

(A) 2 (B) –2 (C) 0

(D) can not be predicted with these informations

14. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be vectors of length 3, 4, 5 respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $|\vec{a} + \vec{b} + \vec{c}|$

(A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$

15. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j}$; $(\vec{a} \wedge \vec{b}) = \pi/2$, $\vec{a}\cdot\vec{c} = 4$, then

(A) $[\vec{a}\,\vec{b}\,\vec{c}]^2 = |\vec{a}|$ (B) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|$

(C) $[\vec{a}\,\vec{b}\,\vec{c}] = 0$ (D) $[\vec{a}\,\vec{b}\,\vec{c}] = |\vec{a}|^2$

16. $(\vec{d} + \vec{a}).(\vec{a} \times (\vec{b} \times (\vec{c} \times \vec{d})))$ simplifies to

(A) $(\vec{b}\cdot\vec{d})[\vec{a}\,\vec{c}\,\vec{d}]$ (B) $(\vec{b}\cdot\vec{c})[\vec{a}\,\vec{b}\,\vec{d}]$

(C) $(\vec{b}\cdot\vec{a})[\vec{a}\,\vec{b}\,\vec{d}]$ (D) none of these

17. Let $\vec{r}$ be a vector perpendicular to $\vec{a} + \vec{b} + \vec{c}$, where $[\vec{a}\,\vec{b}\,\vec{c}] = 2$. If $\vec{r} = \ell(\vec{b}\times\vec{c}) + m(\vec{c}\times\vec{a}) + n(\vec{a}\times\vec{b})$, then $(\ell + m + n)$ is equal to

(A) 2 (B) 1 (C) 0 (D) none of these

18. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar non-zero vectors and $\vec{r}$ is any vector in space, then $(\vec{a}\times\vec{b})\times(\vec{r}\times\vec{c})+(\vec{b}\times\vec{c})\times(\vec{r}\times\vec{a})+(\vec{c}\times\vec{a})\times(\vec{r}\times\vec{b})$ is equal to

(A) $2[\vec{a}, \vec{b}, \vec{c}]\vec{r}$ (B) $3[\vec{a}, \vec{b}, \vec{c}]\vec{r}$

(C) $[\vec{a}, \vec{b}, \vec{c}]\vec{r}$ (D) none of these

19. Given the vertices A (2, 3, 1), B(4, 1, –2), C(6, 3, 7) & D(–5, –4, 8) of a tetrahedron. The length of the altitude drawn from the vertex D is

(A) 7 (B) 9 (C) 11 (D) none of these

20. If a, b, c are pth, qth, rth terms of an H.P. and $\vec{u} = (q - r)\hat{i} + (r - p)\hat{j} + (p - q)\hat{k}$, $\vec{\upsilon} = \frac{\hat{i}}{a} + \frac{\hat{j}}{b} + \frac{\hat{k}}{c}$, then

(A) $\vec{u}, \vec{\upsilon}$ are parallel vectors

(B) $\vec{u}, \vec{\upsilon}$ are orthogonal vectors

(C) $\vec{u}, \vec{\upsilon} = 1$ (D) $\vec{u}\times\vec{\upsilon} = \hat{i} + \hat{j} + \hat{k}$

21. For a non zero vector $\vec{A}$ If the equations $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ hold simultaneously, then

(A) $\vec{A}$ is perpendicular to $\vec{B} - \vec{C}$

(B) $\vec{A} = \vec{B}$ (C) $\vec{B} = \vec{C}$ (D) $\vec{C} = \vec{A}$

22. If the unit vectors $\vec{e}_1$ and $\vec{e}_2$ are inclined at an angle 2θ and $|\vec{e}_1 - \vec{e}_2| < 1$, then for $\theta \in [0, \pi]$, θ may lie in the interval

(A) $\left[0, \frac{\pi}{6}\right)$ (B) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (C) $\left(\frac{5\pi}{6}, \pi\right]$ (D) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

23. A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, $\vec{a}$ has components p + 1 and 1, then

(A) p = 0 (B) p = 1 or p = –1/3

(C) p = –1 or p = 1/3 (D) p = 1 or p = –1

24. Taken on side $\overrightarrow{AC}$ of a triangle ABC, a point M such that $\overrightarrow{AM} = \frac{1}{3}\overrightarrow{AC}$. A point N is taken on the side $\overrightarrow{CB}$ such that $\overrightarrow{BN} = \overrightarrow{CB}$, then for the point of intersection X of $\overrightarrow{AB}$ and $\overrightarrow{MN}$ which of the following holds good ?

(A) $\overrightarrow{XB} = \frac{1}{3}\overrightarrow{AB}$ (B) $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$

(C) $\overrightarrow{XN} = \frac{3}{4}\overrightarrow{MN}$ (D) $\overrightarrow{XM} = 3\overrightarrow{XN}$

25. The volume of the parallelopiped constructed on the diagonals of the faces of the given rectangular parallelopiped is m times the volume of the given parallelopiped. Then m is equal to

(A) 2 (B) 3 (C) 4 (D) none of these

26. If $\vec{a} = \vec{b} + \vec{c}$, $\vec{b} \times \vec{d} = 0$ and $\vec{c} \cdot \vec{d} = 0$ then $\frac{\vec{d}\times(\vec{a}\times\vec{d})}{\vec{d}^2}$ is equal to

(A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{c}$ (D) $\vec{d}$

27. Consider a tetrahedron with faces f_1, f_2, f_3, f_4 Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 and whose directions are perpendicular to these faces in the outward direction. Then

(A) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = 0$ (B) $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$

(C) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ (D) none of these

28. In the isosceles triangle ABC, $|\overrightarrow{AB}| = |\overrightarrow{BC}| = 8$ and a point E divides AB internally in the ratio 1 : 3, then the cosine of angle between $\overrightarrow{CE}$ and $\overrightarrow{CA}$ is (where $|\overrightarrow{CA}| = 12$)

(A) $-\frac{3\sqrt{7}}{8}$ (B) $\frac{3\sqrt{8}}{17}$ (C) $\frac{3\sqrt{7}}{8}$ (D) $-\frac{3\sqrt{8}}{17}$

29. If the vector product of a constant vector $\overrightarrow{OA}$ with a variable vector $\overrightarrow{OB}$ in a fixed plane OAB be a constant vector, then locus of B is
(A) a straight line perpendicular to $\overrightarrow{OA}$
(B) a circle with centre O radius equal to $|\overrightarrow{OA}|$
(C) a straight line parallel to $\overrightarrow{OA}$
(D) none of these

30. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ in terms of θ is equal to
(A) $(1 + \cos\theta)\sqrt{\cos 2\theta}$
(B) $(1 + \cos\theta)\sqrt{1-2\cos 2\theta}$
(C) $(1 - \cos\theta)\sqrt{1+2\cos 2\theta}$ (D) none of these

31. If u and v are unit vectors and θ is the acute angle between them, then 2u × 3v is a unit vector for
(A) Exactly two values of θ
(B) More than two values of θ
(C) No value of θ (D) Exactly one value of θ

32. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals
(A) 0 (B) 1 (C) –4 (D) –2

33. The value of a, for which the points A, B, C with position vectors $2\hat{i} - \hat{j} - \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} - \hat{k}$ respectively are the vertices of a right angled triangle with C = $\pi/2$ are
(A) –2 and –1 (B) –2 and 1
(C) 2 and –1 (D) 2 and 1

34. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is
(A) 10/3 (B) 3/10 (C) $\frac{10}{3\sqrt{3}}$ (D) 10/9

35. Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is $\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector.
(A) (–9, 5, 2) (B) (9, 5, –2)
(C) (9, –5, –2) (D) none of these

36. A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The workdone in standard units by the force is given by
(A) 40 (B) 30 (C) 25 (D) 15

37. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non-coplanar for
(A) all values of λ
(B) all except one value of λ
(C) all except two values of λ
(D) non value of λ

38. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2$, $|\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals
(A) 2 (B) $\sqrt{7}$ (C) $\sqrt{14}$ (D) 14

39. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$, If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals is
(A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $\frac{2\sqrt{2}}{3}$

40. $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$ then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is equal to
(A) 0 (B) –7 (C) 7 (D) 1

41. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}).[(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})]$ equals
(A) 0 (B) $\vec{u} \cdot \vec{v} \times \vec{w}$
(C) $\vec{u} \cdot \vec{w} \times \vec{v}$ (D) $3\vec{u} \cdot \vec{v} \times \vec{w}$

42. Consider points A, B, C and D with position vectors $7\hat{i} + 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. The ABCD is a

(A) square (B) rhombus
(C) rectangle (D) none of these

43. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

44. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$ then $|\vec{w} \cdot \hat{n}|$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

45. If $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{n}$ be a unit vector such that $\vec{b}.\vec{n} = 0$, $\vec{a}.\vec{n} = 0$ then value of $|\vec{c}.\vec{n}|$ is

(A) 1 (B) 3 (C) 5 (D) 2

Sol.

46. If $\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$, then $|\vec{u} \times \vec{v}|$ is equal to

(A) $\sqrt{2(16 - (\vec{a}.\vec{b})^2)}$ (B) $2\sqrt{(16 - (\vec{a}.\vec{b})^2)}$

(C) $2\sqrt{(4 - (\vec{a}.\vec{b})^2)}$ (D) $\sqrt{2(4 - (\vec{a}.\vec{b})^2)}$

47. Equation of a line which passes through a point with position vector $\vec{c}$, parallel to the plane $\vec{r}.\vec{n} = 1$ and perpendicular to the line $\vec{r} = \vec{a} + t\vec{b}$ is

(A) $\vec{r} = \vec{c} + \lambda(\vec{c} - \vec{a}) \times \vec{n}$ (B) $\vec{r} = \vec{c} + \lambda(\vec{a} \times \vec{n})$

(C) $\vec{r} = \vec{c} + \lambda(\vec{b} \times \vec{n})$ (D) $\vec{r} = \vec{c} + \lambda(\vec{b} \times \vec{n})\vec{a}$

48. Points L, M and N lie on the sides AB, BC and CA of the triangle ABC such that ℓ (AL) : ℓ (LB) = ℓ (BM) : ℓ (MC) = ℓ (CN) : ℓ (NA) = m : n, then the areas of the triangles LMN and ABC are in the ratio

(A) $\dfrac{m^2}{n^2}$ (B) $\dfrac{m^2 - mn + n^2}{(m+n)^2}$

(C) $\dfrac{m^2 - n^2}{m^2 + n^2}$ (D) $\dfrac{m^2 + n^2}{(m+n)^2}$

49. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendiuclar to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\dfrac{\pi}{6}$, then

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

(A) 0 (B) 1

(C) $\dfrac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(D) $\dfrac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

50. $[(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}), (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}), (\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})]$ is equal to

(A) $[\vec{a}\vec{b}\vec{c}]^2$ (B) $[\vec{a}\vec{b}\vec{c}]^3$ (C) $[\vec{a}\vec{b}\vec{c}]^4$ (D) none of these

51. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of

$\dfrac{1}{1-a} + \dfrac{1}{1-b} + \dfrac{1}{1-c}$ is equal to

(A) 1 (B) –1 (C) 0 (D) none of these

52. The vectors $\vec{a} = -4\hat{i} + 3\hat{k}, \vec{b} = 14\hat{i} + 2\hat{j} - 5\hat{k}$ are coinitial. The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$ and is having the magnitude $\sqrt{6}$, is

(A) $\hat{i} + \hat{j} + 2\hat{k}$ (B) $\hat{i} - \hat{j} + 2\hat{k}$ (C) $\hat{i} + \hat{j} - 2\hat{k}$ (D) none of these

53. A point taken on each median of a triangle divides the median in the ratio 1 : 3, reckoning from the vertex. Then the ratio of the area of the triangle with vertices at these points to that of the original triangle is

(A) 5 : 13 (B) 25 : 64 (C) 13 : 32 (D) none of these

54. If $\vec{r}.(2\hat{i} + 3\hat{j} - 2\hat{k}) + 3/2 = 0$ is the equation of a plane and $\hat{i} - 2\hat{j} + 2\hat{k}$ is a point, then a point equidistant from the plane on the opposite side is

(A) $\hat{i} + 2\hat{j} + 3\hat{k}$ (B) $3\hat{i} + \hat{j} + \hat{k}$ (C) $3\hat{i} + 2\hat{j} + 3\hat{k}$ (D) $3(\hat{i} + \hat{j} + \hat{k})$

55. If A(1, 1, 1), C(2, −1, 2), the vector equation of the line $\overrightarrow{AB}$ is $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + t(6\hat{i} - 3\hat{j} + 2\hat{k})$ and d is the shortest distance of the point C from $\overrightarrow{AB}$, then
(A) B(6,−3,2) (B) B(5, −4, 1) (C) d= $\sqrt{2}$ (D) d= $\sqrt{6}$

56. If $\vec{b}$ and $\vec{c}$ are any two perpendicular unit vectors and $\vec{a}$ is any vector, then
$(\vec{a}\cdot\vec{b})\vec{b} + (\vec{a}\cdot\vec{c})\vec{c} + \dfrac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}(\vec{b}\times\vec{c})$ is equal to
(A) $\vec{a}$ (B) $\vec{b}$ (C) $\vec{c}$ (D) none of these

57. If $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ are the vertices of a regular plane polygon with n sides and O is its centre then

$\sum_{i=1}^{n-1}(\overline{OA_i} \times \overline{OA_{i+1}})$ equals

(A) (1 − n) $(\vec{OA_2} \times \vec{OA_1})$ (B) (n − 1) $(\vec{OA_2} \times \vec{OA_1})$
(C) n $(\vec{OA_2} \times \vec{OA_1})$ (D) none of these

58. The set of values of 'm' for which the vectors
$\vec{a} = m\hat{i} + (m+1)\hat{j} + (m+8)\hat{k}$,
$\vec{b} = (m+3)\hat{i} + (m+4)\hat{j} + (m+5)\hat{k}$ and
$\vec{c} = (m+6)\hat{i} + (m+7)\hat{j} + (m+8)\hat{k}$ are non-coplanar is
(A) R (B) R − {1} (C) R − {1, 2}(D) ϕ

59. For any four points P, Q, R, S,
$|\overline{PQ}\times\overline{RS} - \overline{QR}\times\overline{PS} + \overline{RP}\times\overline{QS}|$ is equal to 4 times the area of the triangle
(A) PQR (B) QRS (C) PRS (D) PQS

60. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle of $\cos^{-1}\dfrac{11}{14}$ and doubled in magnitude, then it becomes $4\hat{i} + (4x-2)\hat{j} + 2\hat{k}$. The value of 'x' is
(A) $-\dfrac{2}{3}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{1}{3}$ (D) 2

61. Given the three vectors $\vec{a} = -2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 5\hat{j}$ and $\vec{c} = 4\hat{i} + 4\hat{j} - 2\hat{k}$. The projection of the vector $3\vec{a} - 2\vec{b}$ on the vector $\vec{c}$ is
(A) 11 (B) −11 (C) 13 (D) none of these

62. If the acute angle that the vector, $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ makes with the plane of the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is $\cot^{-1}\sqrt{2}$ then
(A) $\alpha(\beta + \gamma) = \beta\gamma$ (B) $\beta(\gamma + \alpha) = \gamma\alpha$
(C) $\gamma(\alpha + \beta) = \alpha\beta$ (D) $\alpha\beta + \beta\gamma + \gamma\alpha = 0$